

PHASES OF THERMAL SUPER YANG-MILLS

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We review the thermodynamics of the confined and unconfined phases of $\mathcal{N} = 4$ super Yang-Mills at large N on a three-sphere, focussing especially on the confinement-deconfinement transition. We determine an N -dependent phase boundary and point out some directions for future work.

1. Confinement in Super Yang-Mills Theory

The correspondence¹ between type IIB string theory in anti-de Sitter space and certain conformal field theories has allowed us to study those gauge theories in regimes that were previously intractable. In this paper, we review some thermodynamic aspects of this duality when the conformal field theory is four-dimensional $\mathcal{N} = 4$ super Yang-Mills with gauge group $SU(N)$ for large rank N .

The duality is holographic; it relates a CFT on the boundary to bulk manifolds with the right symmetries. For the $\mathcal{N} = 4$ $SU(N)$ theory, there are at least two such geometries: $AdS_5 \times S^5$ and a spacetime in which the AdS_5 is replaced with a Hawking-Page black hole (an uncharged black hole in an anti-de Sitter background). These two solutions correspond respectively to the confined and unconfined phases in the gauge theory,² as we shall see. Our aim is to use the correspondence to study the confinement-deconfinement transition in the gauge theory.^a

The gauge theory action is

$$S = \int d^4x \sqrt{-g} \operatorname{Tr} \left(-\frac{1}{4g^2} F^2 + \frac{1}{2} (D\Phi)^2 + \frac{1}{12} R\Phi^2 + \bar{\psi} \not{D} \psi \right). \quad (1)$$

All fields are in the adjoint representation of $SU(N)$. The six scalars, Φ , transform under $SO(6)$ R-symmetry, while the four Weyl fermions, ψ , transform under $SU(4)$, the spin cover of $SO(6)$. The scalars are conformally coupled; otherwise, all fields are massless.

This theory is conformal at the quantum level. In order to induce interesting phase behavior one has to break conformality. We can do this by considering the theory at nonzero temperature, T , on a finite volume three-sphere of radius r_0 . Then

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conformal invariance dictates that thermodynamic properties can depend only on the dimensionless quantity Tr_0 or, in the microcanonical version, Ur_0 where U is the energy of the system. This implies in particular that the infinite volume limit is equivalent to the infinite temperature limit; on R^3 the theory is always in its high temperature phase. Hence to study the confined phase we consider the theory on a finite sphere.

However, “confinement” on S^3 is slightly tricky. One cannot envision a single quark on S^3 as there is nowhere for the flux to go (because $\pi_2(S^3 - \{0\}) \equiv 0$). Instead, we can try to express confinement in terms of temporal Wilson loops (Polyakov loops). A temporal loop, C , traces one end of an open string, whose other end terminates on the stack of N D3-branes, that is, in the bulk. So we should consider all string worldsheets that extend from a timelike circle on the boundary into the bulk. The expectation value over these tube-shaped surfaces serves as an order parameter for the spontaneous breaking of the Z_N center of $SU(N)$:

$$\langle W[C] \rangle = \frac{1}{N} \left\langle \text{Tr } P \exp \oint_C A \right\rangle \sim e^{-\beta F}, \quad (2)$$

where P denotes path-ordering, A is the gauge field, and the trace is in the fundamental representation (to be precise, one should also include the fermions and scalars). The expectation value is taken with respect to the action, Eq. (1). Here we have written it in terms of the free energy. Loosely speaking, adding a Wilson loop is like adding a quark;^{4,5} then since a zero expectation value costs an infinite amount of free energy, it corresponds to confinement. Now the topology of the black hole is $B^2 \times S^3$, while the topology of thermal AdS space is $S^1 \times B^4$. Then the argument goes that, since C belongs to a nontrivial first homology class for the thermal AdS spacetime, it cannot be the boundary of a worldsheet. Hence the expectation value of the Wilson loop is zero and so thermal AdS corresponds to the confined phase. By contrast, for the black hole topology, C could be one boundary of a string worldsheet so in general the Wilson loop has nonzero expectation value. However, there is a subtlety, first noted by Witten.² IIB string theory has an NS-NS two-form that, with vanishing field strength, can be added to the background at no cost in energy. Thus the Wilson loop acquires a periodic phase factor, and summing over all values of the phase gives an expectation value of zero for the black hole geometry too. So Wilson loops do not seem to characterize the phase of the gauge theory on S^3 .

To check that the different gravitational backgrounds indeed correspond to different phases of the gauge theory one could look for other evidence, for example by comparing thermodynamic quantities computed on either side of the correspondence. Unfortunately, at present it is only possible to do direct calculations in the gauge theory at weak coupling. Consider then the free field limit of super Yang-Mills. In the high-energy regime which dominates the state counting, the spectrum of free fields on a sphere is essentially that of blackbody radiation in flat space, with $8N^2$ bosonic ($2N^2$ for the gauge bosons, and $6N^2$ for the scalars) and $8N^2$

fermionic degrees of freedom. The entropy is therefore

$$S_{\text{CFT}} = \frac{2}{3} \pi^2 N^2 V_{\text{CFT}} T_{\text{CFT}}^3 . \quad (3)$$

Since the fields have been taken to be noninteracting, this is obviously the unconfined phase. In the next section, we shall see that anti-de Sitter black holes have almost exactly the same thermodynamics.

2. AdS Black Holes

The line element of the dual ‘‘Schwarzschild’’ black hole⁶ in $AdS_5 \times S^5$ is

$$ds^2 = - \left(1 - \frac{2MG_5}{r^2} + r^2 l^2 \right) dt^2 + \left(1 - \frac{2MG_5}{r^2} + r^2 l^2 \right)^{-1} dr^2 + r^2 d\Omega_3^2 + l^{-2} d\Omega_5^2 , \quad (4)$$

where G_5 is the five-dimensional Newton constant, and l is the inverse radius of both the five-sphere and AdS, so that the five-dimensional cosmological constant is $\Lambda = -6l^2$. The geometry has a Ricci tensor proportional to the metric and a nonvanishing Weyl tensor.

The black hole horizon is at $r = r_+$ where

$$r_+^2 = \frac{1}{2l^2} \left(-1 + \sqrt{1 + 8MG_5 l^2} \right) . \quad (5)$$

When $r_+ l \ll 1$, the black hole could become unstable to localization on the S^5 by an analog of the Gregory-Laflamme mechanism.⁷ A necessary condition⁸ for instability is that the entropy of the localized black hole be larger than the entropy of a black hole uniformly spread over the five-sphere. A straightforward computation then shows that a localization instability could exist for very small black holes with $r_+ l \ll 1$. Here we shall work with stable black holes that have $r_+ l > 1$.

To study the black hole’s thermodynamics, we Euclideanize the metric. The substitution $\tau = it$ makes the metric positive definite and, by the usual removal of the conical singularity at r_+ , yields an inverse temperature of the black hole given by the period of τ :

$$\beta_{\text{BH}} = \frac{2\pi r_+}{1 + 2r_+^2 l^2} . \quad (6)$$

The entropy is

$$S_{\text{BH}} = \frac{A}{4G_5} = \frac{\pi^2 r_+^3}{2G_5} , \quad (7)$$

where A is the ‘‘area’’ (that is, three-volume) of the horizon. The mass above the anti-de Sitter background is

$$U_{\text{BH}} = \frac{3\pi}{4} M = \frac{3\pi}{8G_5} r_+^2 (1 + r_+^2 l^2) . \quad (8)$$

This is the AdS equivalent of the ADM mass, or energy at infinity. (Actually if the black hole is to be considered at thermal equilibrium it should properly be regarded

as being surrounded by a thermal envelope of Hawking particles. Because of the infinite blueshift at the horizon, this envelope contributes a formally infinite energy. Here we shall neglect this infinite energy as unphysical, absorbed perhaps by a renormalization of the Newton constant. But there could be finite regularization-scheme-dependent corrections of order zero in G_5 .)

We can now also write down the free energy:

$$F_{\text{BH}} = \frac{\pi r_+^2}{8G_5} (1 - r_+^2 l^2) . \quad (9)$$

Eqs. (6-9) satisfy the first law of thermodynamics. It is interesting to note that, by formally dividing both the free energy and the mass by an arbitrary volume, one obtains an equation of state:

$$p = \frac{1}{3} \frac{r_+^2 l^2 - 1}{r_+^2 l^2 + 1} \rho , \quad (10)$$

where $p = -F/V$ is the pressure, and ρ is the energy density. In the limit $r_+ l \gg 1$ this equation becomes

$$p = \frac{1}{3} \rho , \quad (11)$$

as is appropriate for the equation of state of a conformal theory. This suggests that if a conformal field theory is to reproduce the thermodynamic properties of this gravitational solution, it has to be in the $r_+ l \gg 1$ limit.

To express the CFT parameters N , T_{CFT} , and V_{CFT} in terms of black hole quantities, we implement holography by taking the physical data for the CFT from the boundary of the black hole spacetime. At fixed $r \equiv r_0 \gg r_+$, the boundary line element tends to

$$ds^2 \rightarrow r_0^2 [-l^2 dt^2 + d\Omega_3^2] , \quad (12)$$

which has a spatial volume of

$$V_{\text{CFT}} = 2\pi^2 r_0^3 . \quad (13)$$

The conformal field theory temperature is the physical temperature at the boundary:

$$T_{\text{CFT}} = \frac{T_{\text{BH}}}{\sqrt{-g_{tt}}} \approx \frac{1}{lr_0} \frac{1 + 2r_+^2 l^2}{2\pi r_+} . \quad (14)$$

To obtain an expression for N , we invoke the AdS/CFT correspondence. This relates N to the radius of S^5 and the cosmological constant:

$$R_{S^5}^2 = \sqrt{4\pi g_s \alpha'^2 N} = \frac{1}{l^2} . \quad (15)$$

Then, since

$$(2\pi)^7 g_s^2 \alpha'^4 = 16\pi G_{10} = 16 \frac{\pi^4}{l^5} G_5 , \quad (16)$$

we have

$$N^2 = \frac{\pi}{2l^3 G_5} . \quad (17)$$

With these substitutions, we see that the CFT entropy in the unconfined phase, Eq. (3), is (almost) the same as the black hole entropy, Eq. (7), in the limit $r_+ l \gg 1$:

$$S_{\text{CFT}} \sim \frac{4}{3} S_{\text{BH}} . \quad (18)$$

Similarly, the red-shifted energy of the conformal field theory matches the black hole mass, modulo a coefficient:

$$U_{\text{CFT}}^\infty = \sqrt{-g_{tt}} \frac{\pi^2}{2} N^2 V_{\text{CFT}} T_{\text{CFT}}^4 \sim \frac{\pi}{2} r_+^4 l^2 \sim \frac{4}{3} U_{\text{BH}} , \quad (19)$$

where U_{CFT}^∞ is the conformal field theory energy red-shifted to infinity, and we have again taken the $r_+ l \gg 1$ limit. At this level, the correspondence only goes through in this high temperature limit. Since the only two scales in the thermal conformal field theory are r_0 and T_{CFT} , high temperature means that $T_{\text{CFT}} \gg 1/r_0$, allowing us to neglect finite-size effects.

The mysterious 4/3 discrepancy in Eqs. (18) and (19) is usually construed to be an artifact of having calculated the gauge theory entropy in the free field limit rather than in the limit of strong 't Hooft coupling, $\lambda \gg 1$, as required by the correspondence; intuitively, one expects the free energy to decrease when the coupling increases. The 4/3 factor was first noticed in the context of D3-brane thermodynamics.⁹ The thermodynamic matching has been extended to rotating black holes^{10,11,12} and their field theory duals, Yang-Mills with angular momentum for which, interestingly, the tantalizing 4/3 factor remains unchanged.¹¹

For our present purposes, the most interesting aspect is the N -dependence of the result. The dependence on N^2 indicates that the conformal field theory is in its unconfined phase; the N^2 species of free gluons make independent contributions to the free energy. We shall see in the next section that the thermodynamics of the confined phase is rather different.

3. A Hot Bath in AdS

Now consider a gas of thermal radiation in anti-de Sitter space. This is dual to the confined phase of super Yang-Mills. (Curiously, attempts to formulate confinement in terms of anti-de Sitter space date back at least to the 1970's.¹³) The thermal gas is necessary to ensure that the energy measured with respect to the AdS background is nonzero. The energy eigenstates of AdS_5 are:¹⁴

$$\Psi_{\omega jmn}(r, t, \theta, \phi, \psi) = N_{\omega j} \exp(-i\omega l t) \sin^j \rho C_{\omega-j-1}^{j+1}(\cos \rho) Y_j^{mn}(\theta, \phi, \psi) , \quad (20)$$

where $C_q^p(x)$ are Gegenbauer polynomials, $Y_j^{mn}(\theta, \phi, \psi)$ are the spherical harmonics in five-dimensional spacetime (with total angular momentum number j), and $\rho \equiv \arctan(r/l)$. Here ω is an integer satisfying the condition $\omega - 1 \geq j \geq |m|, |n|$. Hence

the spectrum is quantized in units of l , the inverse radius of AdS. Since this is also the quantum of excitations of the five-sphere, we should consider thermodynamics over the full ten-dimensional space. The appropriate line element is therefore

$$ds^2 = - (1 + r^2 l^2) dt^2 + (1 + r^2 l^2)^{-1} dr^2 + r^2 d\Omega_3^2 + l^{-2} d\Omega_5^2. \quad (21)$$

To obtain a thermal field theory, we again Euclideanize the metric. The periodicity of $\tau = it$ is then the inverse (asymptotic) temperature, T_{AdS}^{-1} , of the theory; the absence of a horizon means that T_{AdS} is an arbitrary parameter. However, the relevant temperature for thermodynamics in the bulk is not T_{AdS} , but the local, redshifted, temperature:

$$T_{\text{local}} = \frac{T_{\text{AdS}}}{\sqrt{-g_{tt}}} = \frac{T_{\text{AdS}}}{\sqrt{1 + r^2 l^2}}. \quad (22)$$

To calculate thermodynamic quantities we foliate spacetime into (timelike) slices of constant local temperature. Extensive thermodynamic quantities are then computed by adding the contribution of each such hypersurface.

The local energy density of the thermal gas of radiation is

$$\rho_{\text{local}} = \sigma T_{\text{local}}^{10}, \quad (23)$$

where we have neglected infrared effects due to curvature or nonconformality. (This assumes a thermodynamic limit; an exact statistical mechanical computation would be preferable.) Here σ is the ten-dimensional supersymmetric generalization of the Stefan-Boltzmann constant, which is approximated by its flat space value:

$$\sigma = \frac{62}{105} \pi^5, \quad (24)$$

where we have included a factor of 128, the number of massless bosonic physical degrees of freedom of IIB supergravity.

The total “ADM” energy-at-infinity of a gas contained in a ball of radius r_0 is then

$$U_{\text{gas}}^\infty = \sigma \frac{2\pi^5}{l^5} \int_0^{r_0} T_{\text{local}}^{10} \sqrt{-g_{tt}} \sqrt{g_{rr}} r^3 dr \equiv \sigma V_{\text{eff}}(r_0) T_{\text{AdS}}^{10}. \quad (25)$$

Here the additional blueshift factor of $\sqrt{-g_{tt}}$ converts the local (fiducial) energy into an ADM-type energy, comparable to Eq. (8). We have also defined an effective volume,

$$V_{\text{eff}}(r_0) = \frac{2\pi^5}{l^9} \left(\frac{2}{3} - \frac{2 + 3(r_0 l)^2}{3(1 + (r_0 l)^2)^{3/2}} \right), \quad (26)$$

which, as $r_0 \rightarrow \infty$, approaches

$$\frac{4\pi^5}{3l^9}. \quad (27)$$

Thermodynamically, anti-de Sitter space behaves as if it had a finite volume.

Similarly, the other thermodynamic quantities of the thermal bath are

$$F = -\frac{\sigma}{9} V_{\text{eff}} T_{\text{AdS}}^{10}, \quad S = \frac{10}{9} \sigma V_{\text{eff}} T_{\text{AdS}}^9, \quad (28)$$

consistent with the first law of thermodynamics. The absence of a G_5 in the free energy indicates, from the CFT point of view, that the free energy is of order N^0 . This is the confined phase of the theory – the free energy is of order N^0 because the N^2 species of gluons have condensed into hadronic color singlets.

Rewritten in CFT quantities, Eq. (28) implies that $S_{\text{CFT}} \sim r_0^9 T_{\text{CFT}}^9$ since $T_{\text{CFT}} \approx T_{\text{AdS}}/(lr_0)$. The nine-dimensional volume is somewhat puzzling for a three-dimensional gauge theory. It reflects the fact that the QCD (SYM) string is really a type IIB string which naturally lives in nine spatial dimensions. It has been suggested that the extra dimensions in which the open string worldsheet bounded by a Wilson loop can extend are akin to Liouville dimensions.^{1,4,15,16}

4. The Crossover

In the microcanonical approach that we shall now pursue, the contributions to the partition function come from both the black hole and the gas in AdS where the energy of the gas and black hole are taken to be the same. Which of these two thermodynamic phases the system is found in is determined, in the saddle point approximation, by the relative values of the respective Euclidean classical actions:

$$Z(U) = e^{-I_{\text{BH}}(U)} + e^{-I_{\text{gas}}(U)}. \quad (29)$$

The action of the black hole is simply the Einstein-Hilbert action with a negative cosmological constant. This is proportional to the proper volume of spacetime and so needs to be regulated. We will refer to Euclideanized anti-de Sitter space as thermally-identified AdS, to distinguish it from thermal AdS with a gas of radiation. Thermally-identified AdS was chosen as the zero of energy in the black hole mass formula, Eq. (8). We choose it also as the zero of action. A finite black hole action is obtained by subtracting the (also infinite) action for thermally-identified anti-de Sitter space in which the hypersurface at a constant large radius has the same intrinsic geometry as a hypersurface at the same radius in the black hole background.^{2,6} The resultant regularized black hole action is

$$I_{\text{BH}} = -\frac{1}{16\pi G_5} \int d^5x \sqrt{-g} (R + 12l^2) = \frac{\pi^2 r_+^3}{4G_5} \frac{1 - r_+^2 l^2}{1 + 2r_+^2 l^2}. \quad (30)$$

The comparable value of the action for the thermal gas in AdS is just the action of the gas itself, namely F/T :

$$I_{\text{gas}} = -\frac{1}{9} \sigma V_{\text{eff}} T_{\text{AdS}}^9. \quad (31)$$

The qualitative thermodynamic behavior of the system is determined by the phase whose action dominates the partition function Eq. (29). The system can

change phase by tunneling. This is not a phase transition in the Landau sense of a singularity in the free energy; in general, such mathematically strict phase transitions are not possible in finite volume because the partition function is then a finite sum which is therefore analytic in all couplings. Rather, the changes in phase are smooth crossovers that occur with a probability

$$\Gamma \sim \exp(\Delta I) = \exp(-\mathcal{O}(N^2)) , \quad (32)$$

where ΔI is the difference in the Euclidean action between the two phases. As N goes to infinity the crossover does not occur. At the crossover between the two phases, the action for the gas and the black hole are the same. Moreover, energy must be conserved. Thus at the crossover, we have

$$U_{\text{gas}}^{\text{local}} = U_{\text{BH}}^{\text{local}} , \quad I_{\text{gas}} = I_{\text{BH}} . \quad (33)$$

Note that, since the two phases cannot be in physical contact, the physical temperature is not required to be the same for the two phases.

Solving these equations yields N^2 as a function of the dimensionless quantity $x \equiv r_+ l$ at the crossover:

$$N^2 = \frac{31}{2^5 \cdot 3^{13} \cdot 5 \cdot 7} \frac{(1+x^2)^9 \cdot (1+2x^2)^{10}}{(x^2-1)^{10} \cdot x^{12}} . \quad (34)$$

This is the equation we want.³ For given N , it determines the value of x at which the phase changes. Using Eq. (14), we see that x is related to the dimensionless quantity $T_{\text{CFT}} r_0$ (or better, to $U_{\text{CFT}} r_0$) as predicted from conformal invariance. Before discussing the crossover curve, we comment on some of the underlying approximations.

In using Eq. (21), we have omitted the back-reaction of the gas on the metric. Back-reaction can reliably be neglected when the matter term in the parenthesis is smaller than the cosmological term. For an energy density given by Eq. (23) and a total energy matched to that of the black hole phase, Eq. (8), the condition $G_5 \rho < |\Lambda|$ in the $t-t$ Einstein equation amounts to

$$\frac{9\pi}{32} x^2 (1+x^2) l^2 < 6 l^2 , \quad (35)$$

and we see that the matter term becomes dominant at large x , and is not entirely negligible even near $x = 1$. At high temperature, therefore, Eq. (34) becomes unreliable. The exact form of Eq. (34) would also be modified by including the correct Stefan-Boltzmann constant for anti-de Sitter space. More importantly, since the temperature can be of the order of the ground state energy, we need a statistical mechanical derivation of the energy density. And finally, when N is small, the supergravity approximation itself breaks down.

Despite these caveats, Eq. (34) seems to capture the correct qualitative behavior. In Fig. 1, we plot N near the crossover for $x \sim 1$. The region below the crossover

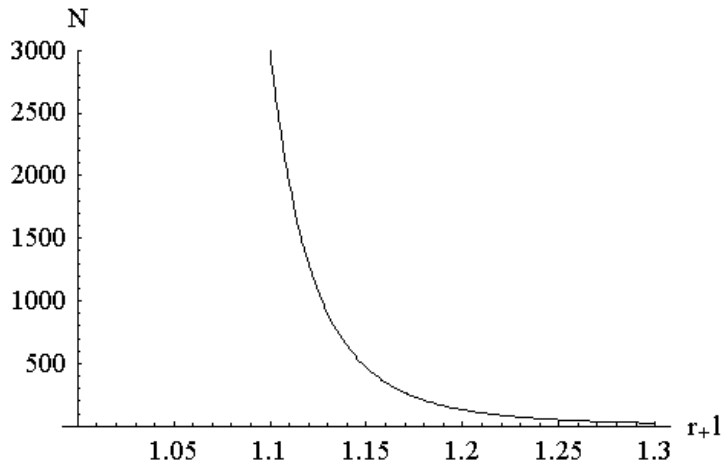


Fig. 1. N vs. $r+l$ at the crossover.

curve is dominated by the confined or AdS gas phase, whereas the region above is dominated by the unconfined or black hole phase. As the temperature increases, the graph confirms our expectation that the gauge theory recovers conformality. As N goes to infinity, we recover the result of Witten,² that the transition occurs at $r+l = 1$. We see that in fact this is a very good approximation for finite (but large) N .

5. Conclusion and Outlook

We have determined the N -dependent curve that separates the confined and unconfined phases of thermal super Yang-Mills theory on a three-sphere, by making use of the AdS/CFT correspondence. A direct derivation of such a phase diagram would have been difficult.

Plenty of work remains to be done. There are several technical assumptions that need to be tightened: the effect of the backreaction of the gas on the metric (if not exactly, then at linear order in G_5), a statistical mechanical (as opposed to thermodynamic) expression for the energy of the gas in AdS, the AdS value of the Stefan-Boltzmann constant, finite one-loop corrections (perhaps using zeta-function regularization) to the black hole background, etc. These technical improvements are important because some of the approximations made here are vulnerable precisely in the regime of greatest interest. The inclusion of other parameters such as rotation or R-charge¹⁷ are obvious generalizations; these more complicated solutions might have phase diagrams with interesting properties. One could also study what happens with different boundaries, or in different dimensions. For AdS_3 , much is known about the corresponding conformal theory; perhaps, this could serve as a test.

The most interesting – if not most promising – direction is to ask whether there are any implications for real multicolor (large N) QCD. Super Yang-Mills differs from ordinary QCD in several ways: it is supersymmetric, of course, also conformal and has extra spin zero and spin one-half adjoint fields. One possible approach to QCD is as follows.² Consider the six-dimensional conformal theory with $(0, 2)$ supersymmetry that is dual to $AdS_7 \times S^4$ (which is the near-horizon geometry of N parallel coincident M5-branes). Now compactify on an S^1 with supersymmetry-breaking anti-periodic boundary conditions for the fermions; since the fermions have no zero modes, they acquire a mass at tree level proportional to the inverse radius of the circle. The mass of the scalars is not protected by any symmetry, so in general we expect them to acquire a mass at one-loop. Compactifying on a second circle then gives ordinary large N pure glue QCD at zero temperature. Unfortunately, the glueballs in this theory typically have an excitation energy of the same order as the Kaluza-Klein modes, suggesting that real QCD may remain out of reach.

It would be interesting to see if our analysis could be pursued in some of these directions.

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